

SURVEY ON PREOPEN SETS*

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Abstract

The aim of this survey article is to cover most of recent research on preopen sets. I try to present majority of the results on preopen sets that I am aware of.

1 Introduction

In 1957, J.D. Weston (*J. London Math. Soc.*, **32** (1957), 342–354) showed that a continuous bijection ϕ on a complete metric space X to a Hausdorff space Y is open if and only if it is nearly open, i.e., if and only if $\phi(G)$ is in the interior of its closure for every open G . By simple derivations from this result, open mapping and continuity theorems in certain spaces were given in 1974 by B.J. Pettis (*Bull. London Math. Soc.*, **6** (1974), 37–41) and applied to groups and linear spaces.

In 1976, T. Byczkowski and R. Pol, (On the closed graph and open mapping theorems, *Bull. Acad. Polon. Sci. Sr. Sci. Math. Astronom. Phys.*, **24** (9) (1976), 723–726) extended Weston’s theorem to the case of a Čech-complete space X , showed that ‘bijection’ cannot be replaced by ‘surjection’, and listed results corresponding to those derived from Weston’s.

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Although not stated, it follows that a continuous homomorphism ϕ on a topological group X to a Hausdorff group is open if $X/\text{Ker}\phi$ is Čech-complete and ϕ maps identity neighborhoods into somewhere dense sets.

The main result of Byczkowski and Pol is that a nearly continuous mapping from Hausdorff space to Čech-complete space is continuous if it has a closed graph and the preimage of every compact set is compact. The interesting proof implicitly establishes an open relations theorem slightly more general than the above extension of Weston's.

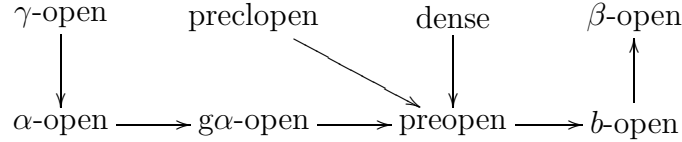
In 1974, P. Mah and S.S. Naimpally, (Open and uniformly open relations, *Bull. Amer. Math. Soc.*, **80** (1974), 1157–1159) announced several open mapping theorems for relations $R \subseteq X \times Y$, where (X, δ) is an Efremovic proximity space and Y a topological space. One of their main results states that, in this setting, R is open if both weakly open and nearly open. These results are used to generalize some theorems of J.D. Weston, J.L. Kelley (General topology, Van Nostrand, Toronto, Ont., 1955) and B.J. Pettis.

Here is the fundamental definition. A subset A of a topological space (X, τ) is called *preopen* or *locally dense* or *nearly open* if $A \subseteq \text{Int}(\text{Cl}(A))$. A set A is called *preclosed* if its complement is preopen or equivalently if $\text{Cl}(\text{Int}(A)) \subseteq A$. If A is both preopen and preclosed, then it is called *preclopen*. The term ‘preopen’ was used for the first time by A.S. Mashhour, M.E. Abd El-Monsef and S.N. El-Deeb (On pre-continuous and weak pre-continuous mappings, *Proc. Math. Phys. Soc. Egypt*, **53** (1982), 47–53), while the concept of a ‘locally dense’ set was introduced by H.H. Corson and E. Michael (Metrizability of certain countable unions, *Illinois J. Math.*, **8** (1964), 351–360).

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called precontinuous if preimages of open sets are preopen (A.S. Mashhour, M.E. Abd El-Monsef and S.N. El-Deeb, same paper as above). Precontinuity was called by V. Ptk nearly continuity (Completeness and open mapping theorem, *Bull. Soc. Math. France*, **86** (1958), 41–74). Nearly continuity or precontinuity is known also as almost continuity (T. Husain, Almost continuous mappings, *Prace Mat.*, **10** (1966) 1–7). Precontinuity was studied for real-valued functions on Euclidean spaces by Blumberg back in 1922 (H. Blumberg, New properties of all real functions, *Trans. Amer. Math. Soc.*, **24** (1922) 113–128).

2 Characterizations and general properties

None of the implications in the following diagram is reversible:



Theorem 2.1 *For a subset A of a topological space (X, τ) the following conditions are equivalent:*

- (1) A is preopen.
- (2) The semi-closure of A is a regular open set.
- (3) A is the intersection of an open set and a dense set.
- (4) A is the intersection of a regular open set and a dense set.
- (5) A is a dense subset of some regular open subspace.
- (6) A is a dense subset of some open subspace.
- (7) A is a dense subset of some preopen subspace.
- (8) A is a preneighborhood of each one of its points.
- (9) $sCl(A) = Int(Cl(A))$.
- (10) There exists a regular open set R containing A such that $Cl(A) = Cl(R)$.

Here are the most fundamental properties of preopen sets:

- *Noiri's Lemma:* If A is semi-open and B is preopen, then $A \cap B$ is semi-open in B and preopen in A . (T. Noiri, Hyperconnectedness and preopen sets, *Rev. Roumaine Math. Pures Appl.*, **29** (4) (1984), 329–334.)

- *Janković and Reilly's Lemma:* Every singleton is either preopen or nowhere dense.
- Arbitrary union of preopen sets is preopen.
- Finite intersection of preopen sets need not be preopen.
- The intersection of a preopen set and an α -open set is a preopen set.
- The intersection $P \cap R$ of a preopen set P and a regular closed (resp. regular open) set R is regular closed (resp. regular open) in the preopen subspace P .

- A set is α -open if and only if it is semi-open and preopen.
- A set is clopen if and only if it is closed and preopen.
- A set is open if and only if it is locally closed and preopen if and only if it is an \mathcal{A} -set and preopen if and only if it is a \mathcal{B} -set and preopen.
- A set is regular open if and only if it is semi-closed and preopen.
- If U is a preopen subspace of a space (X, τ) and V a preopen subset of $(U, \tau|U)$, then V is preopen in (X, τ) .
- If V is preopen such that $U \subseteq V \subseteq \text{Cl}(U)$, then U is also preopen.
- If V is preopen such that $V \subseteq U \subseteq X$, then V is also preopen in $(U, \tau|U)$.
- If A is an α -open subset of a space (X, τ) , then a subset U of A is preopen in $(A, \tau|A)$ if and only if U is preopen in (X, τ) .
- If A is a preopen subset of a space (X, τ) , then for every subset U of A we have $A \cap \text{sCl}(U) = \text{sCl}_A(U)$.
- If A is a preopen subset of a space (X, τ) , then for every subset U of A we have $\text{Int}_A(\text{Cl}_A(U)) = A \cap \text{Int}(\text{Cl}(U))$.
- If P is preopen and S is semi-open, then $P \cap \text{Cl}S = \text{Cl}(P \cap \text{Int}(S)) = \text{Cl}(P \cap S) = \text{Cl}(P \cap \text{Cl}(S)) = \text{Cl}(\text{Int}(\text{Cl}(P) \cap S))$.
- If A is preopen, then $\text{Cl}(A) = \text{Cl}_\theta(A)$.
- $PO(X, \tau) = PO(X, \tau^\alpha)$.
- If P is preopen in (X, τ) and A is an N -subset of $(P, \tau|P)$, then A is an N -subset of (X, τ) .
- Let $(X_i)_{i \in I}$ be a family of spaces and $\emptyset \neq A_i \subseteq X_i$ for each $i \in I$. Then, $\prod_{i \in I} A_i$ is preopen in $\prod_{i \in I} X_i$ if and only if A_i is preopen in X_i for each $i \in I$ and A_i is non-dense for only finitely many $i \in I$.

3 Spaces defined in terms of preopen sets

The following spaces can be characterized (or defined) in terms of preopen sets:

A topology τ on a set X is *extremally disconnected* (M.H. Stone, Algebraic characterizations of special Boolean rings, *Fund. Math.*, **29** (1937), 223–302) if the τ -closure of every member of τ is also in τ .

Theorem 3.1 *For a topological space (X, τ) the following conditions are equivalent:*

- (1) *X is extremally disconnected.*
- (2) *Every regular closed subset of X is preopen.*
- (3) *Every semi-open subset of X is preopen.*
- (4) *The closure of every preopen set is open.*
- (5) *The closure of every preopen set is preopen.*

A nonempty space X is *hyperconnected* if every nonempty open subset of X is dense. A hyperconnected space is called sometimes *irreducible*.

Theorem 3.2 *For a topological space (X, τ) the following conditions are equivalent:*

- (1) *X is hyperconnected.*
- (2) *For the semi-closure $sCl(A)$ of every nonempty preopen subset A of X we have $sCl(A) = X$, i.e., every nonempty preopen subset of X is semi-dense.*
- (3) *Every nonempty preopen subset of X is dense.*
- (4) *If S is semi-open and P is preopen such that $S \cap P = \emptyset$, then $S = \emptyset$ or $P = \emptyset$.*

Recall that a space (X, τ) is called *locally indiscrete* if every open subset of X is closed.

Theorem 3.3 *For a topological space (X, τ) the following conditions are equivalent:*

- (1) *X is locally indiscrete.*
- (2) *Every subset of X is preopen.*
- (3) *Every singleton in X is preopen.*
- (4) *Every closed subset of X is preopen.*

Theorem 3.4 *For a topological space (X, τ) the following conditions are equivalent:*

- (1) *X is locally connected.*
- (2) *Every component of every open subspace is preopen.*

Recall that a space (X, τ) is called *submaximal* if every dense subset of X is open.

Theorem 3.5 *For a topological space (X, τ) the following conditions are equivalent:*

- (1) *X is submaximal.*
- (2) *Every preopen set is open.*

The definition of strongly compact spaces is due to D. Janković, I. Reilly and M. Vamanamurthy (On strongly compact topological spaces, *Questions Answers Gen. Topology*, **6** (1) (1988), 29–40), while the definition of strongly Lindelöf spaces is due to A.S. Mashhour, M.E. Abd. El-Monsef, I.A. Hasanein and T. Noiri (*Delta J. Sci.*, **8** (1984), 30–46).

Theorem 3.6 *For a topological space (X, τ) the following conditions are equivalent:*

- (1) *X is strongly compact (resp. strongly Lindelöf).*
- (2) *Every preopen cover of X has a finite (resp. countable) subcover.*

The definition of countably P -compact is due to S.S. Thakur and P. Paik, (Countably P -compact spaces, *Sci. Phys. Sci.*, **1** (1) (1989), 48–51).

Theorem 3.7 *For a topological space (X, τ) the following conditions are equivalent:*

- (1) *X is countably P -compact.*
- (2) *Every countable family of preopen sets which covers X has a finite subcover.*

The following result is due to M. Ganster, (Preopen sets and resolvable spaces, *Kyungpook Math. J.*, **27** (2) (1987), 135–143).

Theorem 3.8 *For a connected topological space (X, τ) the following conditions are equivalent:*

- (1) *X is resolvable.*
- (2) *The topology on X having the preopen sets of (X, τ) as a subbase is the discrete one.*

In 1987, V. Popa (Properties of H -almost continuous functions, *Bull. Math. Soc. Sci. Math. R.S. Roumanie (N.S.)*, **31** (79) (1987), 163–168) introduced the class of preconnected spaces.

Theorem 3.9 *For a topological space (X, τ) the following conditions are equivalent:*

- (1) *X is preconnected.*
- (2) *X cannot be represented as the disjoint union of two preopen subsets.*

The definition of strongly irresolvable spaces is due to J. Foran and P. Liebnitz (A characterization of almost resolvable spaces, *Rend. Circ. Mat. Palermo*, Serie II, Tomo **XL** (1991), 136–141). They call a space X *strongly irresolvable* if no nonempty open set is resolvable, where a set is resolvable if it is resolvable as a subspace.

Theorem 3.10 *For a topological space (X, τ) the following conditions are equivalent:*

- (1) *X is strongly irresolvable.*
- (2) *Every preopen subset is semi-open.*
- (3) *Every preopen subset is α -open.*

The concept of a p -closed space is due to Abd El-Aziz Ahmed Abo-Khadra, On generalized forms of compactness, Master's Thesis, Faculty of Science, Tanta University, Egypt, 1989.

Theorem 3.11 *For a topological space (X, τ) the following conditions are equivalent:*

- (1) *X is p -closed.*
- (2) *For every preopen cover $\{V_\alpha: \alpha \in A\}$ of X , there exists a finite subset A_0 of A such that $X = \cup\{\text{pcl}(V_\alpha): \alpha \in A_0\}$.*
- (3) *every maximal filter base $\text{pre-}\theta$ -converges to some point of X ,*
- (4) *every filter base $\text{pre-}\theta$ -accumulates at some point of X ,*
- (5) *for every family $\{V_\alpha: \alpha \in A\}$ of preclosed subsets such that $\cap\{V_\alpha: \alpha \in A\} = \emptyset$, there exists a finite subset A_0 of A such that $\cap\{\text{pint}(V_\alpha): \alpha \in A_0\} = \emptyset$.*

Recently, p -closed spaces were extensively studied by me, M. Ganster and T. Noiri. Here are some typical results of our paper:

Theorem 3.12 (1) Let (X, τ) be QHC and strongly irresolvable. Then (X, τ) is p -closed.
(2) Let (X, τ) be a p -closed T_0 space. Then (X, τ) is strongly irresolvable.
(3) If a topological space (X, τ) is p -closed and \aleph_0 -extremally disconnected (resp. extremally disconnected), then (X, τ) is nearly compact (resp. s -closed).

Weak separation axioms in terms of preopen sets were introduced by A. Kar and P. Bhattacharyya, (*Bull. Cal. Math. Soc.*, **82** (1990), 415–422) by replacing the word open with preopen in the classical definitions. Using their technique one can of course define $pre-T_{\frac{1}{2}}$ -spaces as the spaces in which every singleton is either preopen or preclosed. Unfortunately, every spaces is a $pre-T_{\frac{1}{2}}$ -space as shown by H. Maki, J. Umehara and T. Noiri, (Every topological space is $pre-T_{1/2}$, *Mem. Fac. Sci. Kchi Univ. Ser. A Math.*, **17** (1996), 33–42).

N. El-Deeb, I.A. Hasanein, A.S. Mashhour and T. Noiri (On p -regular spaces, *Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.)*, **27** (75) (1983), no. 4, 311–315) introduced the class of p -regular spaces. A topological space X is said to be p -regular if for each closed subset F and each point $x \in X \setminus F$ there exist disjoint preopen sets U and V such that $F \subseteq U$ and $x \in V$. Almost p -regular and p -completely regular spaces were defined by S.R. Malghan and G.B. Navalagi (Almost p -regular, p -completely regular and almost p -completely regular spaces, *Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.)*, **34** (82) (1990), 317–326).

4 Functions defined in terms of preopen sets

Let V (resp. U) be a preopen subset of (Y, σ) (resp. (X, τ)). A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called:

- *Precontinuous* if and only if the preimage of every open subset of Y is preopen in X .
- *Strongly M -precontinuous* (= SMPC) if and only if the preimage of each preopen subset of Y is open in X .
- *Preirresolute* if and only if the preimage of every preopen subset of Y is preopen in X .
- *Faintly precontinuous* if and only if the preimage of every θ -open subset of Y is preopen in X .

- *Strongly faintly precontinuous* if and only if the preimage of every preopen subset of Y is θ -open in X .

- *Weakly continuous* if and only if $f^{-1}(V) \subseteq \text{Int} f^{-1}(\text{Cl}(V))$.
- *Weakly continuous* if and only if $\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V))$.
- *Almost continuous* if and only if $f^{-1}(V) \subseteq \text{Int} f^{-1}(\text{Int}(\text{Cl}(V)))$.
- *Almost open* if and only if $f(U) \subseteq \text{Int}(f(\text{Cl}(U)))$.
- *Almost weakly open* if and only if $f(U) \subseteq \text{Int}(\text{Cl}(f(\text{Cl}(U))))$.
- α -*continuous* if and only if $f(\text{Cl}(U)) \subseteq \text{Cl}(f(U))$.
- *Preclosed* (resp. *preopen*) if and only if the image of every closed (resp. open) set is preclosed (resp. preopen).

5 Generalizations of preopen sets

A subset A of a topological space (X, τ) is called:

- \mathcal{I} -*open* if $A \subseteq \text{Int}(A^*)$ (D. Janković and T.R. Hamlett, Compatible Extensions of Ideals, *Boll. Un. Mat. It.*, **7** (1992), 453–465). Note here that in the case of the minimal ideal, i.e., when $\mathcal{I} = \{\emptyset\}$, the concepts of \mathcal{I} -open and preopen sets coincide.
- γ -*open* if $A \cap S$ is preopen, whenever S is preopen (D. Andrijević and M. Ganster, On PO-equivalent topologies, IV International Meeting on Topology in Italy (Sorrento, 1988), *Rend. Circ. Mat. Palermo (2) Suppl.*, **24** (1990), 251–256). Note that two topologies on a set X are PO-equivalent if their classes of preopen sets coincide.
- *preclopen* if A is both preopen and preclosed (A. Kar and P. Bhattacharyya, (*Bull. Cal. Math. Soc.*, **82** (1990), 415–422).
- *generalized p-closed* (= gp-closed) if $\text{pCl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open (Noiri, T., Maki, H. and Umehara, J., Generalized preclosed functions, *Mem. Fac. Sci. Kochi Univ. Ser. A Math.* **19** (1998), 13–20).
- *regular generalized p-closed* (= rgp-closed) if $\text{pCl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open (Noiri, Almost p-regular spaces and some functions, *Acta Math. Hungar.*, **79** (3) (1998), 207–216).

- *generalized α -closed set* ($= g\alpha$ -closed) if $\text{Cl}_\alpha(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open (H. Maki, R. Devi and K. Balachandran, Generalized α -closed sets in topology, *Bull. Fukuoka Univ. Ed. Part III*, **42** (1993), 13–21).
- *b-open* or *sp-open* if $A \subseteq \text{Cl}(\text{Int}(A)) \cup \text{Int}(\text{Cl}(A))$ (D. Andrijević, On *b*-open sets, *Mat. Vesnik*, **48** (1996), 59–64 and J. Dontchev and M. Przemski, On the various decompositions of continuous and some weakly continuous functions, *Acta Math. Hungar.*, **71** (1-2) (1996), 109–120.)

6 Recent progress in the theory of preopen sets

The following recent papers (a list obtained from MR), more or less deal with (to a certain extent) with preopen sets:

- R.H. Atia, S.N. El-Deeb and I.A. Hasanein, A note on strong compactness and S -closedness, *Mat. Vesnik* **6** (19) (34) (1982), no. 1, 23–28.
- A.S. Mashhour, I.A. Hasanein and S.N. El-Deeb, A note on semicontinuity and precontinuity, *Indian J. Pure Appl. Math.*, **13** (10) (1982), 1119–1123. In this note, the author proves some elementary properties of semicontinuous and precontinuous functions.
- A.S. Mashhour, F.H. Khedr, I.A. Hasanein, and A.A. Allam, S -closedness in bitopological spaces, *Ann. Soc. Sci. Bruxelles Sr. I*, **96** (2) (1982), 69–76. The authors show the relationships of pairwise preopen and α -sets to S -closedness.
- N. El-Deeb, I.A. Hasanein, A.S. Mashhour and T. Noiri, On p -regular spaces, *Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.)*, **27** (75) (1983), no. 4, 311–315. A topological space X is said to be p -regular if for each closed subset F and each point $x \in X \setminus F$ there exist disjoint preopen sets U and V such that $F \subseteq U$ and $x \in V$. The main results of the paper are the following: If a subspace S of a p -regular space is α -open, then it is p -regular. An arbitrary product of p -regular spaces is p -regular. The image of a regular space by a continuous preclosed function with compact point inverses is p -regular.
- A.S. Mashhour, A.A. Allam, F.S. Mahmoud and F.H. Khedr, On supratopological spaces, *Indian J. Pure Appl. Math.*, **14** (4) (1983), 502–510. Results are established for

supratopological spaces which generalize known results about semi-open sets, preopen sets, and α -open sets.

- A.S. Mashhour, M.E. Abd El-Monsef and I.A. Hasanein, On pretopological spaces, *Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.)*, **28** (76) (1984), no. 1, 39–45. A bijective map $f: X \rightarrow Y$ is called a prehomeomorphism whenever $A \subseteq X$ is preopen if and only if $f(A) \subseteq Y$ is preopen. The paper contains some easy remarks on preopen sets, prehomeomorphisms and related concepts. In particular, X is paracompact if and only if every open cover has a locally finite preopen refinement.

- T. Noiri, Hyperconnectedness and preopen sets, *Rev. Roumaine Math. Pures Appl.*, **29** (4) (1984), 329–334. This paper gives a large number of new results and improvements of older results concerning hyperconnected spaces and sets. A sample of the results: If A is preopen in X and B is semi-open in X , and either A or B is hyperconnected in X , then $A \cap B$ is hyperconnected in X . If X is locally hyperconnected (i.e. every $x \in X$ has a base of open hyperconnected neighbourhoods) and if X_0 is a preopen or semi-open subset of X , then X_0 is locally hyperconnected. If a function $f: X \rightarrow Y$ is semicontinuous, then $f(A)$ is hyperconnected for every preopen hyperconnected subset A of X .

- B. Ahmad, and T. Noiri, The inverse images of hyperconnected sets, *Mat. Vesnik*, **37** (2) (1985), 177–181. The following results are proved: (1) If f is an α -continuous function and A is a semi-open hyperconnected set in X , then $f(A)$ is hyperconnected. (2) If f is a semiclosed preserving surjection with preopen hyperconnected point inverses and B is a preopen hyperconnected subset of Y , then $f^{-1}(B)$ is preopen hyperconnected.

- S.F. Tadros, Some special systems of subsets of topological spaces and separation axioms defined by them, *Soobshch. Akad. Nauk Gruzin. SSR*, **118** (1) (1985), 53–56. The first part of the paper contains characterizations of the families of α -open, preopen and β -open subsets from an arbitrary topological space. Further the author establishes some results about the separation properties induced by the families mentioned, linking them to the classical separation axioms (for example, characterizations of Hausdorff, regular and normal spaces in terms of α -sets are given).

- V. Singh, G. Chae and D.N. Misra, (Functions with closed graph and some other related properties, *Univ. Ulsan Rep.*, **17** (1) (1986), 127–131) studied relationships between

functions (with closed graphs, which are continuous or nearly continuous, etc.) and separation axioms satisfied by the range space. A sample theorem of their paper is the following: If $f: X \rightarrow Y$ is a nearly open (= preopen) surjection with a closed graph, then Y is Hausdorff.

- D. Andrijević, A note on preopen sets, Third National Conference on Topology (Italian) (Trieste, 1986), *Rend. Circ. Mat. Palermo (2) Suppl.*, **18** (1988), 195–201. In general, the intersection of two preopen sets need not be preopen, although the union of an arbitrary collection of preopen sets is preopen. This paper gives a set of necessary and sufficient conditions for the intersection of two preopen sets to be preopen.

- M. Ganster, Preopen sets and resolvable spaces, *Kyungpook Math. J.*, **27** (2) (1987), 135–143. This paper provides solutions to the following questions of Katetov. (1) Obtain conditions under which every dense-in-itself set is preopen. (2) Find conditions under which the intersection of any two preopen sets is preopen. (3) Let \mathcal{T}^* denote the topology on X having the collection of all preopen sets in (X, \mathcal{T}) as a subbase. Find conditions under which \mathcal{T}^* is discrete.

Resolvability is the crucial notion in answering these questions. A topological space (X, \mathcal{T}) is defined to be resolvable if there is a subset D of X such that D and $X \setminus D$ are both dense in (X, \mathcal{T}) ; otherwise (X, \mathcal{T}) is called irresolvable.

- D. Andrijević, On the topology generated by preopen sets, *Mat. Vesnik*, **39** (4) (1987), 367–376. The topology mentioned in the title has recently been introduced in a joint paper by the Andrijević and Ganster (same journal **39** (2) (1987), 115–119). The paper is a continuation of the study of this new topology with an emphasis on the properties of its closure operator.

- A.A. Allam, A.M. Zahran and I.A. Hasanein, On almost continuous, S -continuous and set connected mappings, *Indian J. Pure Appl. Math.*, **18** (11) (1987), 991–996. In this paper, the authors strengthen some of the results of P.E. Long and D.A. Carnahan (*Proc. Amer. Math. Soc.*, **38** (1973), 413–418) by using preopenness and openness for both mappings and sets. For example, the theorem that if $f: X \rightarrow Y$ is an open almost continuous mapping then, for each open $V \subseteq Y$, $\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V))$ is improved by letting f be a preopen mapping. The same theorem is also established for a preopen set V with the openness of f

dropped. The authors also improve some of the results of T. Noiri (*J. Korean Math. Soc.*, **16** (2) (1979/80), 161–166) and (*Kyungpook Math. J.*, **16** (2) (1976), 243–246) dealing with weakly continuous and almost continuous mappings.

- V. Popa, Characterizations of H -almost continuous functions, *Glas. Mat. Ser. III*, **22** (42) (1987), no. 1, 157–161. The author characterizes almost continuous functions using preopen and preclosed sets.

- S.S. Thakur, and P. Paik, Locally P -connected spaces, *J. Indian Acad. Math.*, **9** (1) (1987), 34–36. A space X is said to be P -connected if it is not the union of two sets A , B with $A \cap \text{pCl}(B) = \emptyset = \text{pCl}(A) \cap B$. A space X is said to be locally P -connected if for every point $x \in X$ and each open neighbourhood U of x there exists an open P -connected set G such that $x \in G \subseteq U$. The paper contains basic properties of locally P -connected spaces.

- V. Popa, Properties of H -almost continuous functions, *Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.)*, **31** (79) (1987), no. 2, 163–168. The basic properties of almost continuous maps in terms of preopen and preclosed sets are given.

- A.S. Mashhour, A.A. Allam, I.A. Hasanein and K.M. Abd-El-Hakeim, On strongly compactness, *Bull. Calcutta Math. Soc.*, **79** (4) (1987), 243–248.

- T. Noiri, Characterizations of extremally disconnected spaces, *Indian J. Pure Appl. Math.*, **19** (4) (1988), 325–329. In this paper, the author presents several characterizations of extremally disconnected spaces in terms of preopen, semi-open and semi-preopen sets. One of these characterizations says that a space X is extremally disconnected if and only if the closure of every preopen subset of X is preopen. Another one asserts that X is extremally disconnected if and only if for every semi-open set $A \subseteq X$ and every semi-preopen set $B \subseteq X$, the set $A \cap B$ is semi-open.

- M. Ganster, F. Gressl and I.L. Reilly, On a decomposition of continuity, General topology and applications (Staten Island, NY, 1989), 67–72, Lecture Notes in Pure and Appl. Math., **134**, Dekker, New York, 1991. The main result of this paper is that f is continuous if and only if it is precontinuous and weakly \mathcal{B} -continuous. Applications to topological groups and topological vector spaces follow.

- S.S. Thakur and P. Paik, Countably P -compact spaces, *Sci. Phys. Sci.*, **1** (1) (1989), 48–51. A topological space X is defined to be countably P -compact if every countable family

of preopen sets which covers X has a finite subcover. This property implies countable compactness but is not equivalent to it. Characterizations of the notion (similar to those given for countable compactness) are given in terms of countable families of preclosed sets having the countable intersection property and of precluster points of sequences in X . Countable P -compactness is shown to be hereditary for preopen and preclosed subsets and to be preserved by precontinuous, α -open onto mappings.

- M. Ganster, A note on strongly Lindelöf spaces, *Soochow J. Math.*, **15** (1) (1989), 99–104. The author defines X to be d -Lindelöf if each cover of X by dense subsets has a countable subcover, and proves that a space is strongly Lindelöf if and only if it is Lindelöf and d -Lindelöf. Maximal strongly Lindelöf spaces are investigated.

- S.S. Thakur and P. Paik, On semi T_0 , semi T_1 and semi T_2 spaces, *Mathematica (Cluj)*, **31** (54) (1989), 91–94. The authors study in detail the semi- T_0 , semi- T_1 and semi- T_2 spaces, especially the relation between these spaces and preopen semicontinuous functions.

- I. Kupka, On some classes of sets related to generalized continuity, *Acta Math. Univ. Comenian.*, **56/57** (1989), 55–61 (1990). Here the author considers α -sets, semi-open sets and preopen sets. Suitable forms of generalized homeomorphisms related to these classes of sets are considered. The main theorem is as follows: If X, Y are T_1 topological spaces, then the classes of prehomeomorphisms, of semihomeomorphisms and of α -homeomorphisms from X onto Y coincide.

- I.L. Reilly and M.K. Vamanamurthy, On some questions concerning preopen sets, *Kyungpook Math. J.*, **30** (1) (1990), 87–93. In the paper of Mashhour, Abd El-Monsef and El-Deeb, several questions are attributed to Katetov. First, find necessary and sufficient conditions under which every pre-open set is open. Second, find conditions under which every dense-in-itself set is pre-open. Third, find conditions under which the intersection of any two pre-open sets is pre-open. Fourth, find conditions under which the topology generated by the pre-open sets is discrete. The authors solve the first question by showing that every pre-open set is open if and only if every dense set is open (a property considered by Bourbaki). They also provide a partial solution in terms of door spaces. They show, further, that every subset is pre-open if and only if every open set is closed (i.e. if there is

a base which is a partition). The authors partly solve the fourth question by showing that the topology generated by the pre-open sets of any space which can be partitioned into two disjoint dense sets is discrete.

- V. Popa, Some properties of H -almost continuous multifunctions, *Problemy Mat.*, **10** (1990), 9–26. The paper presents various characterizations of almost continuous multivalued maps in the sense of Husain. The results obtained are formulated in terms of preopen and preclosed sets.

- M. Jelić, A decomposition of pairwise continuity, *J. Inst. Math. Comput. Sci. Math. Ser.*, **3** (1) (1990), 25–29. She showed that a function $f(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is pairwise continuous if and only if it is pairwise precontinuous and pairwise LC-continuous.

- M.K. Singal and N. Prakash, Fuzzy preopen sets and fuzzy preseparation axioms, *Fuzzy Sets and Systems*, **44** (2) (1991), 273–281. In this paper, the concept of preopen sets is generalised to the fuzzy setting. Further fuzzy separation axioms are introduced and investigated with the help of fuzzy preopen sets.

- Abdulla S. Bin Shahna, On fuzzy strong semicontinuity and fuzzy precontinuity, *Fuzzy Sets and Systems*, **44** (2) (1991), 303–308. Using the notion of fuzzy sets, Azad, in 1981, introduced and studied the concept of semiopen (semiclosed) sets, semicontinuous mappings, almost continuous mappings, and weakly continuous mappings in the fuzzy setting. In the same spirit, defining fuzzy α -open (α -closed) sets, and fuzzy preopen (preclosed) sets, the authors introduce and make a preliminary study of fuzzy strongly semicontinuous, and fuzzy precontinuous mappings in this paper.

- D. Andrijević, On SPO-equivalent topologies, V International Meeting on Topology in Italy (Italian) (Lecce, 1990/Otranto, 1990), *Rend. Circ. Mat. Palermo (2) Suppl.*, **29** (1992), 317–328. Two topologies on a set X are SPO-equivalent if their classes of semi-preopen sets A coincide. If τ is a topology on X , the topology $\tau_j = \{A \subseteq X: A \cap B \text{ is } \tau\text{-preopen whenever } B \text{ is } \tau\text{-preopen}\}$ is used to identify SPO-equivalent topologies on X ; the class of all topologies on X which are SPO-equivalent to τ has a largest topology τ_R .

- A. Kar and P. Bhattacharyya, Bitopological preopen sets, precontinuity and preopen mappings, *Indian J. Math.*, **34** (3) (1992), 295–309. The concepts of preopen sets, precontinuity and preopen mappings in a bitopological space are introduced in this paper. The

conditions under which the various properties enjoyed by the above concepts in a single topological space can be generalized to a bitopological space are investigated.

- S.S. Thakur and U.D. Tapi, Set P -connected mappings, *Mathematica (Cluj)*, **34** (57) (2) (1992), 183–185. A space X is said to be P -connected between A and B , where $A, B \subseteq X$, if there exists no preclosed preopen $F \subseteq X$ such that $A \subseteq F$ and $F \cap B = \emptyset$. A mapping $f: X \rightarrow Y$ is said to be set P -connected if for all $A, B \subseteq X$, $f(X)$ is P -connected between $f(A)$ and $f(B)$ when X is P -connected between A and B . The authors establish several properties of set P -connected mappings.

- D.S. Janković, and Ch. Konstadilaki-Savvopoulou, On α -continuous functions, *Math. Bohem.*, **117** (3) (1992), 259–270.

- Asit Kumar Sen and P. Bhattacharyya, On preclosed mapping, *Bull. Calcutta Math. Soc.*, **85** (5) (1993), 409–412.

- S. Raychaudhuri and M.N. Mukherjee, On δ -almost continuity and δ -preopen sets, *Bull. Inst. Math. Acad. Sinica*, **21** (4) (1993), 357–366.

- Abdulla S. Bin Shahna, Mappings in fuzzy topological spaces, *Fuzzy Sets and Systems*, **61** (2) (1994), 209–213. In this paper, some characterizations of fuzzy strongly semicontinuous and fuzzy precontinuous mappings are given. The definitions of α -open and preopen mappings are extended to fuzzy sets.

- T. Aho and T. Nieminen, Spaces in which preopen subsets are semiopen, *Ricerche Mat.*, **43** (1) (1994), 45–59. A topological space is called a PS-space if every preopen subset is semiopen; it is called irresolvable if two dense subsets cannot be disjoint. The authors study several equivalent conditions concerning these notions.

- R. Mahmoud and D. Rose, A note on submaximal spaces and SMPC functions, *Demonstratio Math.*, **28** (3) (1995), 567–573. In this paper, the authors give several characterizations of submaximal spaces involving SMPC functions.

- Manindra Chandra Pal and P. Bhattacharyya, Faint precontinuous functions, *Soochow J. Math.*, **21** (3) (1995), 273–289. A function $f: X \rightarrow Y$, where X, Y are topological spaces, is said to be faintly precontinuous if for each open-closed set $W \subseteq Y$ the set $f^{-1}(W)$ is preopen. The paper contains: characterizations of faintly precontinuous functions, relations

to other classes of functions, theorems on preservation of faint precontinuity under operations such as superpositions, restrictions, products. Also the invariance of some separation axioms and properties similar to compactness are considered. Various examples are given.

- Jin Han Park and Yong Beom Park, On sp -regular spaces, *J. Indian Acad. Math.*, **17** (2) (1995), 212–218. In this paper, the authors call a topological space X sp -regular if, for each closed set F and point $x \in X - F$, F and x are contained in disjoint semi-preopen sets. They obtain some equivalent conditions, and study subspaces, products and continuous images of sp -regular spaces.

- S. Raychaudhuri and M.N. Mukherjee, Further characterizations of δ_p -closed spaces, *J. Pure Math.*, **12** (1995), 27–35) continued of the study of δ_p -closedness of topological spaces, a sort of covering property strictly stronger than quasi H -closedness and independent of compactness.

- U.D. Tapi and S.S. Thakur and A. Sonwalkar, Quasi-preopen sets, *J. Indian Acad. Math.*, **17** (1) (1995), 8–12.

- H. Maki, J. Umehara and T. Noiri, Every topological space is pre- $T_{1/2}$, *Mem. Fac. Sci. Kchi Univ. Ser. A Math.*, **17** (1996), 33–42. A subset A of (X, τ) is said to be generalized closed if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) . A subset A of (X, τ) is said to be pre-generalized closed (denoted pg-closed) if $\text{pcl}(A) \subseteq U$ whenever $A \subset U$ and U is preopen in (X, τ) . A space (X, τ) is said to be pre- $T_{1/2}$ if a subset is pg-closed if and only if it is preclosed. In this paper, the authors introduce the concept of a pre-generalized closed set and prove that every topological space is pre- $T_{1/2}$. This is a strengthening of a result by A. Kar and P. Bhattacharyya (*Bull. Calcutta Math. Soc.* **82** (5) (1990), 415–422) that every topological space is pre- T_0 .

- In (D. Somasundaram and V Padmavathy, On generalizations of H -closed spaces, *Indian J. Pure Appl. Math.*, **27** (6) (1996), 557–573.) The authors generalised H -closed spaces by using preopen sets. Analogues of the S -Uryson closed spaces and s -regular closed spaces studied by S.P. Arya and M.P. Bhamini (*Indian J. Pure Appl. Math.*, **15** (1) (1984), 89–98) are considered. An adaptation of the methods employed for HP-closed spaces by T.G. Raghavan and I. Reilly (*Indian J. Math.*, **28** (1) (1986), 75–88) is used to study projective maximum and projective minimum.

- D. Andrijević, On b -open sets, *Mat. Vesnik*, **48** (1-2) (1996), 59–64. The author introduced the class of b -open sets, which is contained in the class of semi-preopen sets and contains all semi-open sets and all preopen sets. It is proved that the class of b -open sets generates the same topology as the class of preopen sets.

- E. Hatir, T. Noiri and S. Yksel, A decomposition of continuity, *Acta Math. Hungar.*, **70** (1-2) (1996), 145–150. The authors define a set S to be a \mathcal{C} -set if $S = U \cap A$, where U is open and A is semi-preclosed. The authors show that a subset of a topological space is open if and only if it is an α -set and a C -set. They define a new class of functions between topological spaces as follows. A function $f: X \rightarrow Y$ is defined to be C -continuous if the preimage (under f) of each open set in Y is a C -set in X . This enables them to provide the following decomposition of continuity: a function is continuous if and only if it is α -continuous and C -continuous.

- Recently, Á. Császár, (Generalized open sets, *Acta Math. Hungar.*, **75** (1-2) (1997), 65–87) provided a unified approach to the study of generalized open (in particular preopen) sets in a topological space, by using monotonic operators $\gamma: \exp(X) \rightarrow \exp(X)$.

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